1) Show that $y = x - x^{-1}$ is a solution of the differential equation $xy' + y = 2x$.

2) Verify that $y = \sin x \cos x - \cos x$ is a solution of the initial-value problem:

$$y' + (\tan x)y = \cos^2 x \qquad y(0) = -1$$

on the interval $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

3) For what nonzero values of k does the function $y = \sin kt$ satisfy the differential equation $y'' + 9y = 0$? For those values of k, verify that every member of the family of functions $y = A\sin kt + B\cos kt$ is also a solution.

4) For what values of r does the function $y = e^{rt}$ satisfy the differential equation $y'' + y' - 6y = 0$?

5) A population is modeled by the differential equation:

$$\frac{dP}{dt} = 1.2P\left(1 - \frac{P}{4200}\right)$$

a) For what values of P is the population increasing?
b) For what values of P is the population decreasing?
c) What are the equilibrium solutions?

6) A function $y(t)$ satisfies the differential equation:

$$\frac{dy}{dt} = y^4 - 6y^3 + 5y^2$$

a) What are the constant solutions of the equation?
b) For what values of y is y increasing?
c) For what values of y is y decreasing?